

K22U 0417

Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2022
(2019 Admission)

CORE COURSE IN MATHEMATICS

Discipline Specific Elective

6B14A MAT : Graph Theory

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any four** questions. Each question carries **one** mark.

1. Define complete graph and complete bipartite graph.
2. State Cayley's theorem on spanning trees.
3. Define Hamiltonian graph.
4. Define Jordan curve.
5. State Euler's formula for connected plane graph.

PART – B

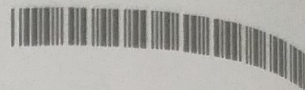
Answer **any eight** questions. Each question carries **two** marks.

6. Explain graph isomorphism with example.

7. Define adjacency matrix of a graph. Draw a graph with adjacency matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

8. Define edge deleted subgraph and vertex deleted subgraph with suitable example.
9. Let G be a k -regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k .
10. Prove that a tree with n vertices has $n - 1$ edges.
11. Prove that an edge e of a graph G is a bridge if e is not a part of any cycle.



12. A graph G is connected, then prove that it has a spanning tree.
13. Explain Chinese post man problem.
14. Describe Konigsberg bridge problem.
15. Define plane graph and planar graph.
16. Define platonic bodies.

PART - C

Answer **any four** questions. **Each** question carries **four** marks.

17. Prove that in a graph G there are even number of odd vertices.
18. Prove that if G is a connected graph, then G is a tree if and only if every edge of G is a bridge.
19. Prove that if G is an acyclic graph with n vertices k connected components, then G has $n - k$ edges.
20. If degree of each vertex of a graph G is at least two, then prove that G contains cycles.
21. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.
22. Prove that the complete graph K_5 is non planar.
23. State Kuratowski's theorem for planar graphs. Using the theorem show that Petersen graph is non planar.

PART - D

Answer **any two** questions. **Each** question carries **six** marks.

24. a) Define complement of a graph with example.
b) What is self - complementary graphs ? Give example.
c) Prove that if G is self-complementary graph with n vertices, then n is either $4t$ or $4t + 1$.
25. Let G be a non empty graph with at least two vertices, prove that G is bipartite if and only if it has no odd cycles.
26. Prove that G is an Euler graph if and only if every vertex is of even degree.
27. Let G be a simple planar graph with n vertices, e edges where $n \geq 3$, then prove that $e \leq (3n - 6)$.