K22U 0417

Reg. No. : ……………………………

Name : ……………………………

# VI Semester B.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2022
## (2019 Admission)
## CORE COURSE IN MATHEMATICS
## Discipline Specific Elective
## 6B14A MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

### PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Define complete graph and complete bipartite graph.
2. State Cayley's theorem on spanning trees.
3. Define Hamiltonian graph.
4. Define Jordan curve.
5. State Euler's formula for connected plane graph.

### PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Explain graph isomorphism with example.
7. Define adjacency matrix of a graph. Draw a graph with adjacency matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

8. Define edge deleted subgraph and vertex deleted subgraph with suitable example.
9. Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.
10. Prove that a tree with n vertices has n – 1 edges.
11. Prove that an edge e of a graph G is a bridge if e is not a part of any cycle.

P.T.O.

12. A graph G is connected, then prove that it has a spanning tree.
13. Explain Chinese post man problem.
14. Describe Konigsberg bridge problem.
15. Define plane graph and planar graph.
16. Define platonic bodies.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Prove that in a graph G there are even number of odd vertices.
18. Prove that if G is a connected graph, then G is a tree if and only if every edge of G is a bridge.
19. Prove that if G is an acyclic graph with n vertices k connected components, then G has n – k edges.
20. If degree of each vertex of a graph G is at least two, then prove that G contains cycles.
21. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.
22. Prove that the complete graph $K_5$ is non planar.
23. State Kuratowski's theorem for planar graphs. Using the theorem show that Petersen graph is non planar.

PART – D

Answer **any two** questions. **Each** question carries **six** marks.

24. a) Define complement of a graph with example.
    b) What is self – complementary graphs ? Give example.
    c) Prove that if G is self-complementary graph with n vertices, then n is either 4t or 4t + 1.
25. Let G be a non empty graph with at least two vertices, prove that G is bipartite if and only if it has no odd cycles.
26. Prove that G is an Euler graph if and only if every vertex is of even degree.
27. Let G be a simple planar graph with n vertices, e edges where $n \geq 3$, then prove that $e \leq (3n - 6)$.

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